AD-A117 492 OKLAHOMA UNIV NORMAN F/G 13/13

THREE-DIMENSIONAL FINITE-ELEMENT ANALYSIS OF LAYERED COMPOSITE --ETC(U)

JUL 82 W C CHAO, N S PUTCHA, J N REDDY N00014-78-C-0647

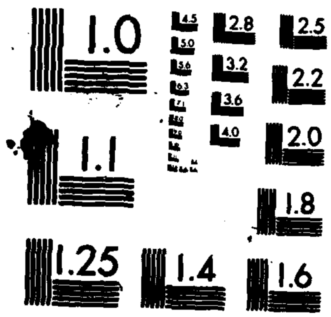
UNCLASSIFIED VPI-E-82-19 NL

END

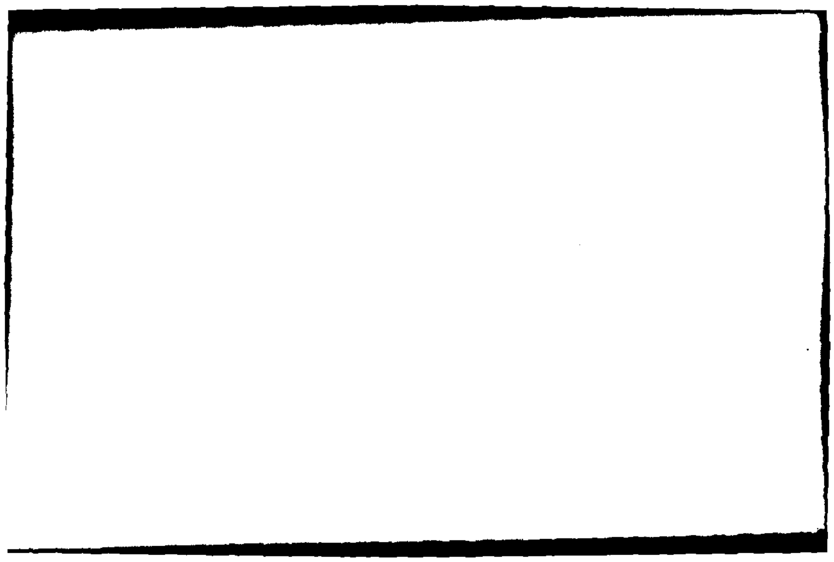
DATE FILMED

9 82

DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A



(11)

Department of the Navy
OFFICE OF NAVAL RESEARCH
Engineering Mechanics Division
Arlington, Virginia 22217

Contract N00014-78-C-0647
Project NR o64-609
Technical Report No. 29

Report VPI-E-82.19

THREE-DIMENSIONAL FINITE-ELEMENT ANALYSIS OF LAYERED COMPOSITE STRUCTURES

W. C. Chao, N. S. Putcha, and J. N. Reddy
Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia
24061

July, 1982

Part I:

GEOMETRICALLY NONLINEAR ANALYSIS OF LAYERED COMPOSITE SHELLS

W. C. Chao
Graduate Research Assistant

J. N. Reddy
Professor, Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University, Blacksburg, VA 24061

ABSTRACT

Two kinds of finite-element analyses are presented for the investigation of the large deformation phenomenon in laminated composite structures, especially shells. The first kind of finite-element analysis employs the general incremental variational formulation as well as the total Lagrangian description of motion. The element adopted is the "degenerate" three-dimensional element. The second analysis adopts a formulation based on deformable shell theory, and the plate-bending element is used. Numerical results for bending are presented for plate and shell structures of isotropic as well as orthotropic composition. The resuts are in good agreement with results available in the literature.

INTRODUCTION

The shell is a common load-carrying element in most industrial equipment, especially in aerospace, nuclear and offshore engineering. Although linear models suffice in many aspects of engineering, a linear analysis of shells yields results too gross and inaccurate to be useful. The nonlinear behavior of structures should be taken into account if a thin shell structure design, dictated by economic considerations, is desired. There exist a number of shell finite elements in the literature. A discussion of various elements is given by Gallagher [1].

The degenerate three-dimensional element, originally proposed by Ahmad, Irons and Zienkiewicz [2] for linear analysis of thin and thick shells, was applied to problems dealing with the geometrical and material nonlinearities in [3] and [4]. It is now believed that this element performs well in shell analysis. Worsak [5] proposed a bilinear degenerate element, and Worsak, et al. [6] developed an eight-node element that has relative displacement degrees-of-freedom in place of the cumbersome finite rotations. The derivation of both elements was based on the concept of the degenerate 3-dimensional element model. Gallagher [7], and Brebbia and Connor [8] extended this concept to the geometrically nonlinear analysis of shells. The degenerate 3-D element is usually accompanied by incremental formulations in the analysis of geometric and material nonlinearities in problems of structures; various incremental variational formulations for the total Lagrangian as well as the updated Lagrangian approach are presented in [9,10].

In modeling laminated composite-material shells, Dong, Pister and Taylor [11] formulated a theory of thin shells laminated of anisotropic material, which is an extension of the theory developed by Stavsky [12] for laminated anisotropic plates based on Love's first approximation theory of shells. Venkatesh and Rao [13] used a doubly curved quadrilateral element to model laminated shells of revolution; there are 12 unknown displacements at each node. Noor [14-16] used a mixed formulation to compare several elements in the linear analysis of laminated shells. Hsu, Reddy and Bert [17] employed the shear-deformable element proposed by Reddy [18] to analyse shells composed of bimodular materials. Noor [19] employed the classical shell element that includes the geometric nonlinearity to solve a spherical shell problem. A degenerate element associated with the updated Lagrangian formulation is presented in [20] to solve a cylinder problem.

The present paper deals with the geometrically nonlinear analysis of layered composite shells, using a 3-D degenerate element. For the shear deformable shell element description, see Reddy [21,22].

A 3-D DEGENERATE ELEMENT FOR LAYERED COMPOSITE SHELLS

Consider the motion of a material body in a Euclidean space, as shown in Figure 1. Here C^t denotes the configuration of the body at time t. To facilitate the description of the kinematics of the continuum at any time, the initial state of the body can be choosen as the reference configuration to which all subsequent configurations are referred to; this is known as the total Lagrangian description. If the coordinates are updated at each incremental time-step, and the kinematics of the continuum is described in terms of the current configuration, the description is called the updated Lagrangian description. It is noted that a load step is used instead of the time step if we are solving the static bending problem.

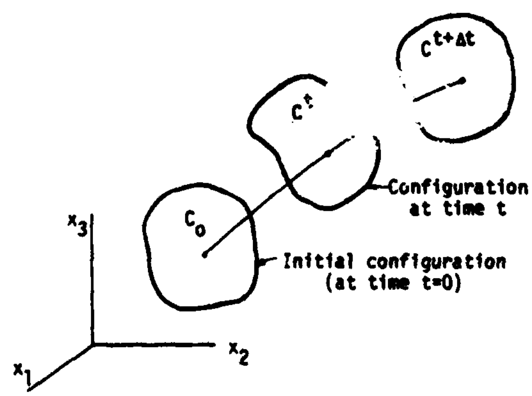


Figure 1. Description of the motion of a a continuum

Invoking the principle of virtual displacements, one can express the equilibrium equation of the body at time $t + \Delta t$ as:

$$\int_{V_0} {}^{t+\Delta t}_{0}S_{ij}\; {}^{t+\Delta t}_{0}\delta\epsilon_{ij}\, dV_0 = \int_{A_0} {}^{t+\Delta t}_{0}T_i\, \delta u_i\, dA + \int_{V_0} {}^{t+\Delta t}_{0}F_i\, \delta u_i\, dV \quad (1)$$

$$= {}^{t+\Delta t}_{0}\delta W$$

where the left superscript refers to the configuration at which the quantity is calculated; the left subscript is the configuration to which the calculation is referred, S_{ij} is the second Piola-Kirchhoff stress tensor and ϵ_{ij} is the Green-Lagrangian strain tensor defined as

$${}^{t+\Delta t}_{0}\epsilon_{ij} = \frac{1}{2}\left({}^{t+\Delta t}_{0}u_{i,j} + {}^{t+\Delta t}_{0}u_{j,i} + {}^{t+\Delta t}_{0}u_{k,i}\; {}^{t+\Delta t}_{0}u_{k,j}\right), \quad (2)$$

T_i and F_i denote the components of surface and body forces, and V_0 and A_0 the volume and area, respectively, of the body in the initial configuration. Using the relations

$${}^{t+\Delta t}_{0}S_{ij} = {}^{t}_{0}S_{ij} + {}_{0}S_{ij} \quad (3a)$$

and

$${}^{t+\Delta t}_{0}u_{i} = {}^{t}_{0}u_{i} + {}_{0}u_{i}, \quad (3b)$$

where S_{ij} and ${}_0u_i$ are the incremental stress and displacement components, we write

$${}^{t+\Delta t}_{0}\epsilon_{ij} = {}^{t}_{0}\epsilon_{ij} + {}_{0}\epsilon_{ij}, \quad (4)$$

where

$${}_{0}\epsilon_{ij} = {}_{0}e_{ij} + {}_{0}\eta_{ij} \quad (5a)$$

$${}_{0}e_{ij} = \frac{1}{2}\left({}_{0}u_{i,j} + {}_{0}u_{j,i} + {}^{t}_{0}u_{k,i}\; {}_{0}u_{k,j} + {}_{0}u_{k,i}\; {}^{t}_{0}u_{k,j}\right) \quad (5b)$$

$${}_{0}\eta_{ij} = \frac{1}{2}\, {}_{0}u_{k,i}\; {}_{0}u_{k,j} \quad (5c)$$

The linear constitutive relations are given by

$${}_{0}S_{ij} = C_{ijk\ell}\; {}_{0}\epsilon_{k\ell}. \quad (6)$$

The equilibrium equation (1) then becomes

$$\int_{V_0} {}_{0}C_{ijk\ell}\; {}_{0}e_{k\ell}\; {}_{0}\delta e_{ij}\, dV_0 + \int_{V_0} {}^{t}_{0}S_{ij}\; {}_{0}\delta\eta_{ij}\, dV_0$$

$$= {}^{t+\Delta t}_{0}\delta W - \int_{V_0} {}^{t}_{0}S_{ij}\; {}_{0}\delta e_{ij}\, dV_0 \quad (7)$$

which is a nonlinear equation in terms of the incremental displacements u, v and w. For the degenerate element (see [23]) this equation can be cast in terms of the middle surface nodal variables u_0, v_0, w_0, α and β. In matrix form, we have

$$([K_L] + [K_{NL}])\{u\} = -\{R\} + \{F\} \quad (8)$$

where $[K_L]$, $[K_{NL}]$ and $\{R\}$ are the linear stiffness matrix, nonlinear stiffness matrix and unbalanced terms, respectively; $\{F\}$ is the forcing term from external loading. To solve these equations, the Newton-Raphson or the modified Newton-Raphson method can be used. If the $[K_{NL}]$ is updated after each iteration, one has the Newton-Raphson method. When the $[K_{NL}]$ is kept constant during each step, one has the Modified Newton-Raphson method.

NUMERICAL RESULTS

The finite-element formulation presented herein includes the transverse shear effect, although we have assumed that the straight line normal to the mid-plane before deformation remains straight after deformation. The reduced integration technique is used in the numerical integration. Due to the biaxial symmetry in the problems considered, only a quarter domain is considered. A 2x2 mesh of 8-node and 9-node elements are employed in this analysis.

PROBLEM 1: Laminated Composite Plates

A simply supported, 2-layer crossply square plate made of graphite-epoxy material subjected to a uniformly distributed load is analysed. The material properties are

$$E_1/E_2 = 40\ ,\ G_{12}/E_2 = 0.5\ ,\ \nu_{12} = 0.25 \quad (9)$$

and the boundary conditions are

$$w = \psi_x = 0 \quad \text{at } y = 0, a,$$
$$w = \psi_y = 0 \quad \text{at } x = 0, a, \quad (10)$$

The numerical solutions for both elements are presented in Table 1. It is clear that the DST predicts lower deflections than the 3-D theory.

Table 1. Comparison of the nonlinear center-deflection (w/h) of Problem 1 obtained by the 3-D degenerate and shell elements.

$\hat{P}$	1.0	2.0	3.0	4.0	5.0	6.0
3-D	1.0414	1.6431	2.0604	2.3871	2.6595	2.8955
DST	1.0754	1.6042	1.9971	2.3008	2.5560	2.7716

$\hat{P} = [P_0(a/h)^4/E_2] \times 10^{-2}$,

3-D = degenerate 3-D element

DST = deformable shell (2-D) theory [21]

PROBLEM 2: Isotropic Cylindrical Panel With Uniform Loading

Consider the circular cylindrical panel shown in Figure 2. The panel is clamped along all four edges, and is subjected to uniform normal loading. A 3x3 mesh of 9-node elements is used. The dimensional center deflections obtained by both elements are listed in Table 2. Once again we note that the DST element predicts lower deflections.

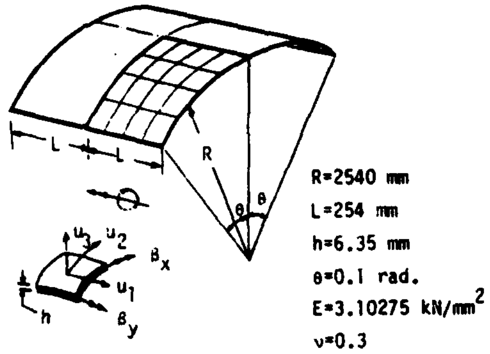


Figure 2. Cylindrical panel used in Problems 2 and 3

Table 2. Comparison of the nonlinear center-deflection w (in mm) of Problem 2 obtained by the 3-D degenerate element and shell element

P (kN/m^2)	1.0	1.5	1.75	2.0	2.25	2.50
3-D	1.4117	2.6131	6.3165	7.6686	8.6041	9.3426
DST	1.3827	3.8828	5.9449	7.5132	8.546	9.2949

PROBLEM 3: <u>Hinged Circular Cylindrical Panel With Pin Load</u>

Using the same geometry of the panel in Problem 2, but with thickness 12.7mm and boundary conditions *such that the curved edges are hinged and the straight* edges are free. A concentrated normal loading is applied at the center. Table 3 contains the numerical results:

Table 3: Comparison of the nonlinear center-deflection w (in mm) in Problem 3 obtained by the 3-D degenerate element and shell element

P (kN)	0.5	1.0	1.5	1.7	2.0	2.2
3-D	1.3784	2.9786	4.9390	5.8965	7.719	9.7715
DST	1.3707	2.9591	4.9088	-	7.627	9.4931

PROBLEM 4: <u>Laminated Composite Spherical Shell With Uniform Loads</u>

Consider a 9-layered cross-ply laminated shell of graphite-epoxy (see Figure 3). The four edges are simply supported; i.e.,

$$\begin{aligned} u = w = \psi_x = 0 \text{ at } y = 0,\ a, \\ v = w = \psi_y = 0 \text{ at } x = 0,\ a. \end{aligned} \tag{11}$$

A uniform inward loading is applied on the shell. The numerical results are tabulated in Table 4. The *present results compare very well with those of Noor* [19]. In this case, the DST element predicts larger deflections while Noor's mixed element predicts lower deflections than those predicted by the 3-D degenerate element.

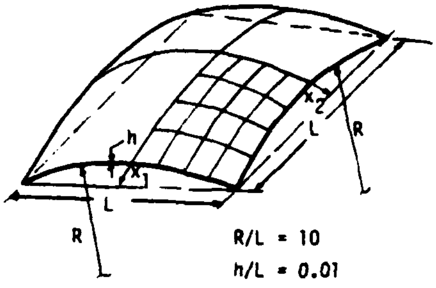


Figure 3. Spherical shell used in Problem 4

Table 4. Comparison of the nonlinear center-deflection (w/h) for Problem 4 obtained by various investigators

$\hat{P}$	2	4	5	6	8	10
DST	0.70918	1.8591	-	2.8299	3.4240	3.8804
3-D	0.69053	1.7607	-	2.7649	3.4170	3.8897
NOOR	0.6806	1.741	2.305	-	3.436	3.916

PROBLEM 5: <u>Laminated Cylindrical Panel With Uniform Loading</u>

The same shell geometry as in Problem 3, but with 2 layers [0°/90°] is used. The material is graphite-epoxy and all four edges are simply supported, i.e.,

$$\begin{aligned} u = w = \psi_x = 0 \text{ at } y = 0, a, \\ v = w = \psi_y = 0 \text{ at } x = 0, a. \end{aligned} \tag{12}$$

The results are compared in Table 5.

Table 5. Comparison of the nonlinear center-deflection (w/h) of Problem 5 obtained by the 3-D degenerate element and shell element.

$\hat{P} = \left(\frac{P_o a^4}{E_2 h^4}\right)$	50	100	200	300	350
DST	0.6326	1.2533	1.9739	2.3988	-
3-D	0.6014	1.2340	1.9812	2.4317	2.471

ACKNOWLEDGEMENTS

The results presented in this paper were obtained during an investigation supported, in part, by the Structural Research Section of NASA/Lewis through Grant NAG 3-208 and Engineering Mechanics Division of ONR through Contract N00014-78-C-0647. The encouragement and support of this work by Dr. Christo C. Chamis (NASA/Lewis) is gratefully acknowledged.

REFERENCES

1. Gallagher, R. H., "Problems and Progress in Thin Shell Finite Element Analysis," Finite Elements for Thin Shells, D. G. Ashwell and R. H. Gallagher eds., Wiley, New York, 1976, pp. 1-14.

2. Ahmad, S., Irons, B. M., and Zienkiewicz, O. C., "Analysis of Thick and Thin Shell Structures by Curved Elements," Int. J. Num. Meth. Engng., Vol. 2, 1970, pp. 419-451.

3. Krakeland, B., "Nonlinear Analysis of Shells Using Degenerated Isoparametric Elements," Int. Conf. on Finite Elements in Nonlinear Solid and Struct. Mech. at Geilo, Norway, Aug. 1977, pp. 265-284.

4. Ramm, E., "A Plate Shell Element for Large Deflections and Rotations," Formulations and Computational Algorithms in Finite Element Analysis, Bathe, Oden and Wunderlick (eds.), MIT Press, Cambridge, 1977.

5. Worsak, K. N., "A Simple and Efficient Finite Element for General Shell Analysis," Int. J. Num. Meth. Engng., Vol. 14, 1979, pp. 179-200.

6. Worsak, K. N., Taylor, R. L., and Hughes, T. J. R., "A Large Deformation Formulation for Shell Analysis by the Finite Element Method," Comp. and Struct., Vol. 13, 1981, pp. 19-27.

7. Gallagher, R. H., "Finite Elements in Nonlinear Mechanics," Int. Conf. on Finite Elements in Nonlinear Solid and Struct. Mech. at Geilo, Norway, 1977.

8. Brebbia, C. and Connor, J., "Geometrically Nonlinear Finite Element Analysis," Proc. ASCE, J. Engng. Mech. Div., 1969, pp. 463-483.

9. Horrigmoe, G. and Bergan, P. G., "Incremental Variational Principles and Finite Element Models for Nonlinear Problems," Comp. Meth. in Appl. Mech. and Engng., Vol. 7, 1977, pp. 201-217.

10. Wunderlich, W., "Incremental Formulations for Geometrically Nonlinear Problems," Formulations and Computational Algorithms in Finite Element Analysis, Bathe, Oden and Wunderlich, eds., 1977, pp. 193-239.

11. Dong, S. B., Pister, K. S., and Taylor, R. L., "On the Theory of Laminated Anisotropic Shells and Plates," J. of Aerospace Science, Vol. 29, 1962, pp. 969-975.

12. Stavsky, Y., "Bending and Stretching of Laminated Anisotropic Plates," J. of the Engng. Mech. Div., Vol. 87, EM6, 1961, p. 31.

13. Venkatesh, A. and Rao, K. P., "A Doubly Curved Quadrilateral Finite Element for the Analysis of Laminated Anisotropic Thin Shells of Revolution," Computers and Structures, Vol. 12, 1980, pp. 825-832.

14. Noor, A. K., and Mathers, M. D., "Shear-Flexible Finite Element Models of Laminated Composite Plates and Shells," NASA TN D-8044, Langley Research Center, Hampton, VA.

15. Noor, A. K., and Camin, R. A., "Symmetry Considerations for Anisotropic Shells," Comp. Meth. in Appl. Mech. and Engng., Vol. 9, 1976, pp. 317-335.

16. Noor, A. K., and Anderson, C. M., "Mixed Isoparametric Finite Element Models of Laminated Composite Shells," Comp. Meth. in Appl. Mech. and Engng., Vol. 11, 1977, pp. 255-280.

17. Hsu, Y. S., Reddy, J. N. and Bert, C. W., "Thermoelasticity of Circular Cylindrical Shells Laminated of Bimodular Composite Materials," J. of Thermal Stresses, Vol. 4, No. 2, 1981, pp. 155-177.

18. Reddy, J. N., "A Penalty Plate Bending Element for the Analysis of Laminated Anisotropic Composite Plates," Int. J. for Num. Meth. in Engng., Vol. 15, 1980, pp. 1187-1206.

19. Noor, A. K., and Hartley, S. J., "Nonlinear Shell Analysis Via Mixed Isoparametric Elements," Comp. and Struct., Vol. 7, 1977, pp. 615-626.

20. Chang, T. Y., and Sawamiphakdi, K., "Large Deformation Analysis of Laminated Shells by Finite Element Method," Comp. and Struct., Vol. 13, 1981, pp. 331-340.

21. Reddy, J. N., "A Finite-Element Analysis of Large-Deflection Bending of Laminated Anisotropic Shells," Nonlinear Finite Element Analysis of Plates and Shells, AMD-Vol. 48, The American Society of Mechanical Engineers, New York, 1981, pp. 249-264.

22. Reddy, J. N., "Bending of Laminated Anisotropic Shells by a Shear Deformable Finite Element," Fibre Science and Technology, 1982 (to appear).

23. Chao, W. C., and Reddy, J. N., "Large Deformation Bending of Laminated Composite Shells Using a 3-D Degenerate Element," Report, Department of Engineering Science and Mechanics, VPI&SU, Blacksburg, VA, 1982.

Part II:
THREE-DIMENSIONAL FINITE-ELEMENT ANALYSIS OF LAYERED COMPOSITE PLATES

N. S. Putcha
Graduate Research Assistant

J. N Reddy
Professor, Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University, Blacksburg, VA 24061

ABSTRACT

Results of a preliminary study of the three-dimensional, geometrically nonlinear, finite-element analysis of the bending of laminated anisotropic composite plates are presented. Individual laminae are assumed to be homogeneous, transversely isotropic, and linearly elastic. The investigation utilizes a fully three-dimensional isoparametric finite element with eight nodes (i.e., linear element) and 24 degrees of freedom (three displacement components per node). Numerical results of the linear analysis are compared with the exact solutions obtained by Pagano. The present results converged to the exact solution as mesh is refined. Due to lack of space, results of the nonlinear analysis were not included here; the results will appear elsewhere.

INTRODUCTION

The development of simple finite elements for the analysis of plates has been a subject of continued interest among researchers of computational solid mechanics. Most of the currently available elements are based on two-dimensional thin-plate theory in which the plane sections normal to the midsurface before deformation are assumed to remain straight and normal to the midsurface after deformation (i.e., transverse shear strains are assumed to be zero) and thick-plate theory that accounts for constant or linear variations of transverse shear strains through the thickness. With the increased use of advanced fiber composite materials in aerospace and automotive structures, a more accurate prediction of stresses in composite plates is needed. Composite materials exhibit higher stiffness-to-weight ratios and increased corrosion resistance compared to isotropic materials. The anisotropic material properties of layered composites can be varied by varying the fibre orientation and stacking sequence. While this feature gives the designer an added degree of flexibility, the stiffness mismatch of the orthotropic layers bonded together with different fiber orientations leads to interlaminar stresses in the vicinity of free edges. For certain stacking sequences, loading, and boundary conditions interlaminar stresses can be so large that they dictate the design of the structure. An accurate prediction of the interlaminar stresses is possible only when a three-dimensional theory is used.

Previous analyses of layered composite plates can be divided into two groups: (i) analyses based on a lamination theory, and (ii) analyses based on a fully three-dimensional theory. The lamination theory is an extension of the classical plate theory (CPT) or the Reissner-Mindlin shear-deformable plate theory (SDT) to layered composite plates. The first lamination theory is apparently due to Reissner and Stavsky [1]. Yang, Norris, and Stavsky [2] presented a generalization of the Reissner-Mindlin thick-plate theory for homogeneous, isotropic plates to arbitrarily laminated anisotropic plates. Whitney and Pagano [3] (also see Reddy and Chao [4]) presented closed-form solutions to the theory when applied to certain cross-ply and angle-ply rectangular plates. Reddy [5] presented a finite-element analysis of the lamination theory. A higher-order lamination theory that accounts for a cubic variation (as opposed to linear in [2-5]) of the inplane displacements and quadratic variation of the transverse displacement through the thickness is presented by Lo, Christensen, and Wu [6], and hybrid-stress finite-element analysis of the theory is presented by Spilker [7]. A generalization of the von Karman nonlinear plate theory of isotropic plates to anisotropic plates is due to Ebcioglu [8]. Chia and his colleagues [9,10] presented approximate numerical solutions, using the perturbation method and the Galerkin method, of the von Karman equations associated with layered composite plates. Recently, Reddy and Chao [11,12] presented finite-element analyses of the same equations.

In the lamination theory it is assumed that the laminate is in a state of plane stress (an assumption carried from the plate theory) and integrals through the thickness of a laminate are equal to the sum of integrals through the thickness of individual laminae. These assumptions lead to inaccurate prediction of interlaminar stresses at the free edges; away from free edges the lamination theory solutions are very accurate. Thus the lamination theory is not accurate in a boundary layer region which extends inward from the edge to a distance approximately equal to the laminate thickness.

The fully three-dimensional theory is nothing but the linear elasticity theory of a three-dimensional solid. Pipes and Pagano [13] and Pipes [14] used a finite-difference technique to solve the quasi-three-dimensional elasticity equations for laminates (also see Hsu and Herakovich [15]). Lin [16], Dana [17], and Dana and Barker [18] used a cubic, three-dimensional, isoparametric element with 72 degrees of freedom to analyze laminated plates. The numerical results in these studies agree very well with those of Pagano [19,20]. The studies were confined to geometrically (as well as materially) linear analyses. Recently, Griffin, Kamat, and Herakovich [21] investigated three-dimensional inelastic (i.e., material nonlinearity) finite-element analysis of laminated composites, while Brockman [22] developed a finite-element program (called MAGNA) for three-dimensional, geometrically, nonlinear, analysis of isotropic plates.

The present study is motivated by the lack of finite-element results for three-dimensional, geometrically nonlinear, analysis of layered composite plates. The present paper gives a finite-element formulation of the nonlinear equations governing a

linearly elastic, 3-D continuum. Numerical results of the linear analysis are presented for several problems and the linear formulation is validated by comparing the present results with those of Pagano [19,20] and others. Results of the nonlinear analysis will appear elsewhere.

Governing Equations

Consider a laminate (Ω) composed of N orthotropic layeres with axes of elastic symmetry parallel to the plate axes. The laminate is subjected to normal traction $\hat{t}_3 = q(x_1,x_2)$ at its upper surface (i.e., $x_3 = h/2$). The constitutive equations for any layer are given by

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0\\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0\\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & Q_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & Q_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & Q_{66}\end{bmatrix} \begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{13}\\ 2\varepsilon_{12}\end{Bmatrix} \tag{1}$$

and the governing field equations are given by (in the absence of body forces)

$$[\sigma_{ij}(\delta_{mi} + u_{m,i})]_{,j} = 0 \tag{2}$$

where σ_{ij} and ε_{ij} are the rectangular Cartesian components of the stress and strain tensors, respectively, and Q_{ij} are the material properties of a lamina in the laminate coordinates. In (2) the summation convention on repeated indices is used. The strain-displacement equations are given by

$$\varepsilon_{ij} = \frac{1}{2}[u_{i,j} + u_{j,i} + u_{m,i}u_{m,j}] \tag{3}$$

To complete the description of the field equations, Eqs. (1)-(3) should be adjoined by boundary conditions. At any point of the boundary of the plate one should specify one of the following two types of boundary conditions:

(i) essential (or geometric) boundary conditions

$$u_i = \hat{u}_i \tag{4a}$$

(ii) natural (or dynamic) boundary conditions

$$t_m \equiv n_j\sigma_{ij}(\delta_{mi} + u_{m,i}) = \hat{t}_m. \tag{4b}$$

Finite-Element Formulation

Here we present a displacement finite-element model of the equations (1)-(4). To this end we construct the variational formulation of the equations over an arbitrary element $\Omega^{(e)}$ of the finite-element mesh. We have (see [23, p. 382])

$$0=\int_{\Omega^{(e)}}\{\delta u_{m,j}[\sigma_{ij}(\delta_{mi}+u_{m,i})]dx_1dx_2dx_3 - \int_{\Gamma^{(e)}} t_m\delta u_m ds \tag{5}$$

Here δ denotes the variational symbol, and σ_{ij} is given in terms of u_i via Eqs. (1) and (3):

$$\sigma_{ij} = \bar{Q}_{ijmn}\varepsilon_{mn} = \frac{1}{2}\bar{Q}_{ijmn}[u_{m,n}+u_{n,m}+u_{k,m}u_{k,n}] \tag{6}$$

where

$$\bar{Q}_{ijmn} = \begin{cases} Q_{ij} & \text{, if } i = m,\ j = n\\ 0 & \text{, otherwise.}\end{cases} \tag{7}$$

Next, we assume that the displacements u_m are interpolated by expressions of the form,

$$u_m = \sum_{\alpha=1}^{8} u_m^\alpha\psi_\alpha \qquad (m = 1,2,3) \tag{8}$$

where $\psi_\alpha(x,y,z)$ ($\alpha = 1,2,\ldots,8$) are the trilinear interpolation functions of the eight-node isoparametric element in three dimensions. Substituting Eq. (8) into (5), we obtain

$$\sum_{n=1}^{3}\sum_{\beta=1}^{8} K_{\alpha\beta}^{mn}u_n^\beta + F_\alpha^m = 0,\ (m=1,2,3;\ \alpha=1,2,\ldots,8), \tag{9}$$

where

$$K_{\alpha\beta}^{mn} = \int_{\Omega^{(e)}} \frac{\partial\psi_\alpha}{\partial x_j}\{\sigma_{nj}(\delta_{mn} + \sum_{\beta=1}^{8} u_m^\beta \frac{\partial\psi_\beta}{\partial x_n})\}dx_1dx_2dx_3$$

$$F_\alpha^m = \int_{\Gamma^{(e)}} t_m\psi_\alpha ds\ ,\ (j,m,n = 1,2,3) \tag{10}$$

Every isoparametric finite element $\Omega^{(e)}$ of the finite-element mesh can be generated from the master element (which is a cube of side 2) via the transformation (see Figure 1)

$$x_i = x_i(\xi_1,\xi_2,\xi_3) = \sum_{\alpha=1}^{8} x_i^\alpha\psi_\alpha(\xi_1,\xi_2,\xi_3) \quad (i = 1,2,3) \tag{11}$$

where x_i^α are the global coordinates of the element nodes. Therefore, the integrals in Eq. (10) can be transformed to the master element and evaluated numerically using the Gauss quadrature. The transformation of $\frac{\partial\psi_\alpha}{\partial x_i}$ to $\frac{\partial\psi_\alpha}{\partial\xi_i}$ is performed as follows (see Reddy [24]):

$$\underset{3\times1}{\left\{\frac{\partial\psi_\alpha}{\partial x_i}\right\}} = [J]^{-1}\underset{3\times1}{\left\{\frac{\partial\psi_\alpha}{\partial\xi_i}\right\}}$$

$$dx_1dx_2dx_3 = (\det[J])d\xi_1d\xi_2d\xi_3 \tag{12a}$$

where

$$[J]_{3\times 3} = \underset{3\times 8}{\begin{bmatrix} \frac{\partial\psi_1}{\partial\xi_1} & \frac{\partial\psi_2}{\partial\xi_1} & \cdots & \frac{\partial\psi_8}{\partial\xi_1} \\ \frac{\partial\psi_1}{\partial\xi_2} & \frac{\partial\psi_2}{\partial\xi_2} & \cdots & \frac{\partial\psi_8}{\partial\xi_2} \\ \frac{\partial\psi_1}{\partial\xi_3} & \frac{\partial\psi_2}{\partial\xi_3} & \cdots & \frac{\partial\psi_8}{\partial\xi_3} \end{bmatrix}} \underset{8\times 3}{\begin{bmatrix} x_1^1 & x_2^1 & x_3^1 \\ x_1^2 & x_2^2 & x_3^2 \\ \vdots & \vdots & \vdots \\ x_1^8 & x_2^8 & x_3^8 \end{bmatrix}} \tag{12b}$$

For example, consider

$$\int_{\Omega^{(e)}} \frac{\partial\psi_\alpha}{\partial x_i}\frac{\partial\psi_\beta}{\partial x_j}dx_1dx_2dx_3 = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} F_{\alpha\beta}^{ij}(\xi_1,\xi_2,\xi_3)d\xi_1 d\xi_2 d\xi_3$$

$$= \sum_{I=1}^{m}\sum_{J=1}^{m}\sum_{K=1}^{m} F_{\alpha\beta}^{ij}(P_I,P_J,P_K)W_I W_J W_K \tag{13}$$

where P_I and W_I are the Gauss points and weights, respectively, and the integrand $F_{\alpha\beta}^{ij}$ ($i,j = 1,2,3$; $\alpha,\beta = 1,2,\ldots,8$) is given by

$$F_{\alpha\beta}^{ij} = \left(\sum_{k=1}^{3} J_{ik}^{*}\frac{\partial\psi_\alpha}{\partial\xi_k}\right)\left(\sum_{\ell=1}^{3} J_{j\ell}^{*}\frac{\partial\psi_\beta}{\partial\xi_\ell}\right)\det[J] \tag{14}$$

$J_{k\ell}^{*}$ being the elements of the inverse of the Jacobian matrix, $[J]$. This procedure can be implemented on a digital computer, and the element coefficient matrices in Eq. (10) can be evaluated numerically. The 1x1x1 Gauss quadrature was used to evaluate the coefficients of Q_{44}, Q_{55}, and 2x2x2 Gauss rule was used to integrate all other terms.

Since the coefficient matrix [K] depends on the unkown solution vector $\{u\}$, one should employ an iterative solution procedure to solve the finite-element equations. Here we use the Picard type iterative technique, which begins with an assumed displacement field (usually, set to zero to obtain the linear solution) to compute [K] at the beginning of the first iteration. In subsequent iterations, the solution obtained from the previous iterations is used to compute [K]. The iteration is continued until the solutions obtained in two consecutive iterations differ by a preassigned error margin (say, 10^{-4}).

Discussion of the Numerical Results

Here we present results of the linear analysis of three test cases that have been analyzed by Pagano [20]:

1. A symmetric three-ply square ($a = b$) laminate with direction 1 of material principal axes coincides with the x_1-direction in the outer layers and direction 2 is parallel to the x_1-axis in the center layer. The layers are of equal thickness.

2. The same lamination geometry as in (1) above, except that $b = 3a$.

3. A square ($a = b$) sandwich plate with the thickness of the face sheets equal to h/10, where h is the total thickness of the laminate.

In all three cases, the loading is assumed to be sinusoidal (the coordinate system is taken at the center of the plate).

$$\hat{t}_3 \equiv q(x_1,x_2) = q_0 \cos\frac{\pi x_1}{a}\cos\frac{\pi x_2}{b} \tag{15}$$

and the boundary conditions are of the simply supported type which allow normal displacement on the boundary, but prevent tangential displacement. For a quarter plate these imply

$$\begin{aligned} &\text{At } x_1 = a/2: \ \hat{u}_2 = \hat{u}_3 = \hat{t}_x = 0 \\ &\text{At } x_1 = 0: \ \hat{u}_1 = 0,\ \hat{t}_y = 0,\ \hat{t}_z = 0 \\ &\text{At } x_2 = b/2: \ \hat{u}_1 = \hat{u}_3 = \hat{t}_y = 0 \\ &\text{At } x_2 = 0: \ \hat{u}_2 = 0,\ \hat{t}_x = 0,\ \hat{t}_z = 0. \end{aligned} \tag{16}$$

The material of the laminae in cases 1 and 2 and face sheets in case 3 is assumed to have the following values for the material coefficients:

$$\begin{aligned} &E_1 = 1.724 \times 10^8 \text{ kN/m}^2 \ (25 \times 10^6 \text{ psi}) \\ &E_2 = E_3 = 6.89 \times 10^6 \text{ kN/m}^2 \ (10^6 \text{ psi}) \\ &G_{12} = G_{13} = 3.45 \times 10^6 \text{ kN/m}^2 \ (0.5 \times 10^6 \text{ psi}) \\ &G_{23} = 13.78 \times 10^6 \text{ kN/m}^2 \ (0.2 \times 10^6 \text{ psi}) \\ &\nu_{12} = \nu_{31} = \nu_{32} = 0.25 \end{aligned} \tag{17}$$

The properties of the core material in case 3 are given by

$$\begin{aligned} &E_1 = E_2 = 0.275 \times 10^6 \text{ kN/m}^2 \ (0.04 \times 10^6 \text{ psi}) \\ &E_3 = 3.45 \times 10^6 \text{ kN/m}^2 \ (0.5 \times 10^6 \text{ psi}) \\ &G_{13} = G_{23} = 0.413 \times 10^6 \text{ kN/m}^2 \ (0.06 \times 10^6 \text{ psi}) \\ &G_{12} = 0.11 \times 10^6 \text{ kN/m}^2 \ (0.016 \times 10^6 \text{ psi}) \\ &\nu_{12} = \nu_{31} = \nu_{32} = 0.25 \end{aligned} \tag{18}$$

The following normalizations are used to present the results:

$$(\bar{\sigma}_1,\bar{\sigma}_2,\bar{\sigma}_{12}) = \frac{h^2}{q_o a^2}(\sigma_1,\sigma_2,\sigma_{12})$$

$$(\bar{\sigma}_{23},\bar{\sigma}_{13}) = \frac{h}{q_o a}(\sigma_{23},\sigma_{13})$$

$$\bar{u}_1 = \frac{E_2 h^2}{q_o a^3}u_1 \ , \quad \bar{u}_3 = \frac{(E_2 h^3 u_3)10^2}{q_o a^4} \tag{19}$$

Table 1 contains a comparison of the present finite-element solution (for various meshes) for the transverse center deflection and stresses with the exact solution of Pagano [20] for Case 1. As the mesh is refined, the finite-element solution converges toward the exact solution. Further mesh refinements were not done due to the limitations on computational resources (i.e., storage, computational time, etc.). From an examination of the results one can see that the finite-element results agree better with the exact solution for increasing values of span-to-depth ratio (a/h). The transverse shear stresses are less accurately predicted than the other stresses.

Similar information is presented in Tables 2 and 3 for Cases 2 and 3, respectively. Figure 2 contains

a plot of the transverse shear stress distribution over the thickness for a/h = 10. The plot agrees, qualitatively, very well with the plot presented in Figure 7 of Reference 20 for a/h = 4.

Acknowledgements

The work reported herein is supported, in part, by Structural Mechanics Sections of NASA/Lewis (under grant NAG 3-208) and Office of Naval Research (under contract N00014 78-C-0647). The support is gratefully acknowledged.

References

1. Reissner, E., and Stavsky, Y., "Bending and stretching of certain types of heterogeneous aeolotropic elastic plates," J. Appl. Mech., Vol. 28, 1961, pp. 402-408.

2. Yang, P. C., Norris, C. H., and Stavsky, Y., "Elastic wave propagation in heterogeneous plates," Int. J. Sol. Struct., Vol. 2, 1966, pp. 665-684.

3. Whitney, J. M., and Pagano, N. J., "Shear deformation in heterogeneous anisotropic plates," J. Appl. Mech., Vol. 37, 1970, pp. 1031-1036.

4. Reddy, J. N., and Chao, W. C., "A comparison of closed-form and finite element solutions of thick laminated anisotropic rectangular plates," Nuclear Engineering and Design, Vol. 64, 1981, pp. 153-167.

5. Reddy, J. N., "A penalty plate-bending element for the analysis of laminated anisotropic composite plates," Int. J. Num. Meth. Engng., Vol. 15, 1980, pp. 1187-1206.

6. Lo, K. H., Christensen, R. M., and Wu, E. M., "A higher-order theory of plate deformation: part 1, homogeneous plates; part 2, laminated plates," J. Appl. Mech., Vol. 44, 1977, pp. 663-668, 669-676.

7. Spilker, R. L., "High order three-dimensional hybrid-stress elements for thick-plate analysis," Int. J. of Num. Meth. in Engng., Vol. 17, 1981, pp. 53-69.

8. Ebcioglu, I. K., "A large deflection theory of anisotropic plates," Ingenieur-Archiv, Vol. 33, 1964, pp. 396-403.

9. Chia, C. Y., and Prabhakara, M. K., "Large deflection of unsymmetric cross-ply plates," J. Mech. Engng. Sci., Vol. 18, 1976, pp. 179-183.

10. Chia, C. Y., and Prabhakara, M. K., "A general mode approach to non-linear flexural vibrations of laminated rectangular plates," J. Appl. Mech. Vol. 45, 1978, pp. 623-628.

11. Reddy, J. N., and Chao, W. C., "Non-linear bending of thick rectangular, laminated composite plates," Int. J. Non-linear Mechanics, Vol. 16, 1981, pp. 291-301.

12. Reddy, J. N. and Chao, W. C., "Nonlinear oscillations of laminated, anisotropic, rectangular plates," J. Appl. Mech. Vol. 49, 1982, pp. 396-402.

13. Pipes, R. B., and Pagano, N. J., "Interlaminar stresses in composite laminates under uniform axial extension," J. Composite Materials, Vol. 4, 1970, pp. 538-548.

14. Pipes, R. B., "Interlaminar stresses in composite laminates," AFML-TR-72-18, 1972.

15. Hsu, P. W., and Herakovich, C. T., "Edge effects in angle-ply composite laminates," J. Composite Materials, Vol. 11, 1977, pp. 422-428.

16. Lin, F. T., "The finite element analysis of laminated composites," Ph.D. Thesis, Virginia Polytechnic Institute and State University, December 1971.

17. Dana, J. R., "Three dimensional finite element analysis of thick laminated composites - including interlaminar and boundary effects near circular holes," Ph.D. Thesis, Virginia Polytechnic Institute and State University, August 1973.

18. Dana, J. R., and Barker, R. M., "Three dimensional analysis for the stress distribution near circular holes in laminated composites," Research report No. VPI-E-74-18, Virginia Polytechnic Institute and State University, August 1974.

19. Pagano, N. J., "Exact solutions for composite laminates in cylindrical bending," J. Composite Materials, Vol. 3, 1969, pp. 398-411.

20. Pagano, N. J., "Exact solutions for rectangular bidirectional composites and sandwich plates," J. Composite Materials, Vol. 4, 1970, pp. 20-35.

21. Griffin, O. H., Kamat, M. P., and Herakovich, C. T., "Three dimensional inelastic finite element analysis of laminated composites," Research report No. VPI-E-80.28, Virginia Polytechnic Institute and State University, November 1980.

22. Brockman, R. A., "MAGNA: A finite element system for three-dimensional nonlinear static and dynamic structural analysis," Computers and Structures, Vol. 13, 1981, pp. 415-423.

23. Reddy, J. N. and Rasmussen, M. L., Advanced Engineering Analysis, Wiley-Interscience, New York, 1982.

24. Reddy, J. N., The Finite Element Method: A Variational Approach, McGraw-Hill, New York, 1983 (to appear).

25. Panda, S. C., and Natarajan, R., "Finite element analysis of laminated composite plates," Int. J. Num. Meth. Engng., Vol. 14, 1979, pp. 69-79.

26. Mawenya, A. S., and Davies, J. D., "Finite element bending analysis of multilayer plates," Int. J. Num. Meth. Engng., Vol. 8, 1974, pp. 215-225.

Table 1. Comparison of the finite-element results with the exact solution of Case 1

$\frac{a}{h}$	SOURCE	Normalized Centre Deflection $\bar{u}_3(0,0,0)$	NORMALIZED STRESS $\bar{\sigma}$ $\bar{\sigma}_1(0,0,\frac{\pm h}{2})$	$\bar{\sigma}_2(0,0,\frac{\pm h}{6})$	$\bar{\sigma}_{13}(\frac{a}{2},0,0)$	$\bar{\sigma}_{23}(0,\frac{b}{2},0)$	$\bar{\sigma}_{12}(\frac{a}{2},\frac{b}{2},\pm\frac{h}{2})$
2	Exact: Pagano [20]		1.436 −0.938	0.669 −0.742	0.164	0.2591	−0.0859 0.0702
	Present: FEM						
	2x2x3	7.5170	0.9880 −0.6332	0.5493 −0.6396	0.1350	0.1871	−0.05412 0.04977
	3x3x3	7.1955	1.025 −0.6680	0.5834 −0.6837	0.1520	0.2022	−0.05843 0.05362
	4x4x3	7.0893	1.037 −0.6803	0.5953 −0.6990	0.1584	0.2081	−0.05999 0.05503
10	Exact: Pagano [20]		±0.590	0.285 −0.288	0.357	0.1228	∓0.0289
	Present: FEM						
	2x2x3	0.7685	0.5349 −0.5337	0.2756 −0.2791	0.3099	0.07761	−0.0214 0.02156
	3x3x3	0.76189	0.5636 −0.5627	0.2832 −0.2870	0.3407	0.08255	−0.02300 0.02318
	4x4x3	0.75985	0.5738 −0.5730	0.285 −0.288	0.3521	0.08439	−0.02360 0.02378
	Reddy and Chao [4]	0.669	±0.5499	±0.252	0.406	0.091	∓0.0250
100	Exact: Pagano [20]		±0.539	±0.181	0.395	0.0828	∓0.0213
	Present: FEM						
	2x2x3	0.41971	±0.4948	±0.1670	0.3425	0.05057	−0.01577 0.01578
	3x3x3	0.43217	±0.5249	±0.1770	0.3735	0.05539	∓0.01732
	4x4x3	0.43639	±0.5356	±0.1805	0.3844	0.05716	∓0.01789
	Reddy and Chao [4]	0.434	±0.535	±0.179	0.422	0.070	∓0.0212
	Classical Plate Theory		±0.539	±0.180	0.395	0.0823	∓0.0213

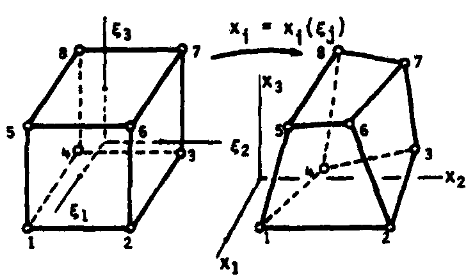


Figure 1. Transformation of the master element to an arbitrary element of a finite-element mesh

Table 2. Comparison of the finite-element results with the exact solution of Case 2

$\frac{a}{h}$	SOURCE	Normalized Centre Deflection $\bar{u}_3(0,0,0)$	NORMALIZED STRESS $\bar{\sigma}$ $\bar{\sigma}_1(0,0,\frac{\pm h}{2})$	$\bar{\sigma}_2(0,0,\frac{\pm h}{6})$	$\bar{\sigma}_{13}(\frac{a}{2},0,0)$	$\bar{\sigma}_{23}(0,\frac{b}{2},0)$	$\bar{\sigma}_{12}(\frac{a}{2},\frac{b}{2},\frac{\pm h}{2})$
4	Exact: Pagano [20]	2.82	1.14 -1.10	0.109 -0.119	0.351	0.0334	-0.0269 0.0281
	Present: FEM						
	2x2x3	3.0723	0.9577 -0.9151	0.1040 -0.1249	0.2963	0.01355	-0.02027 -0.02241
	3x3x3 (Meshes)	2.9554	0.9897 -0.9493	0.1043 -0.1238	0.3286	0.01425	-0.02105 0.02334
	4x4x3	2.9170	1.0001 -0.9614	0.1048 -0.1231	0.3405	0.01452	-0.02134 0.02368
10	Exact: Pagano [20]	0.919	0.726 -0.725	0.0418 -0.0435	0.420	0.0152	-0.0120 0.0123
	Present: FEM						
	2x2x3	0.9208	0.6451 -0.6440	0.0386 -0.0420	0.3586	0.006974	-0.008716 0.009113
	3x3x3	0.9107	0.6776 -0.6768	0.0393 -0.0432	0.3929	0.007613	-0.009319 0.009744
	4x4x3	0.9076	0.6892 -0.6885	0.0396 -0.0435	0.4054	0.007851	-0.009543 0.009978
	Reddy [5]	0.802	±0.603	±0.0364			∓0.0102
	Panda and Natarajan [25]	0.752	±0.653	±0.0367			∓0.0105
	Mawenya and Davies [26]	1.141	±0.685				∓0.0141
100	Exact: Pagano [20]	0.508	±0.624	±0.0253	0.439	0.0108	∓0.0083
	Present: FEM						
	2x2x3	0.48186	±0.5628	0.02355 -0.02351	0.3759	0.005567	-0.006038 0.006042
	3x3x3	0.49683	±0.5978	0.02495 -0.02499	0.4102	0.006208	-0.006639 0.006643
	4x4x3	0.50191	±0.6101	0.02526 -0.02550	0.4227	0.006445	-0.006861 0.006865
	5x5x3	0.50424	±0.6159	±0.02527	0.4287	0.006556	-0.006965 0.006970
	Reddy [5]	0.506	±0.603	±0.0253			∓0.0080
	Panda and Natarajan [25]	0.505	±0.654	±0.0261			∓0.0086
	Mawenya and Davies [26]	0.510	±0.638				∓0.0085
	Classical Plate Theory	0.503	±0.623	±0.0252	0.440	0.0108	∓0.0083

Table 3. Comparison of the finite-element results with the exact solution of Case 3

$\frac{a}{h}$	SOURCE	Normalized Centre Deflection $\bar{u}_3(0,0,0)$	NORMALIZED STRESS $\bar{\sigma}$ $\bar{\sigma}_1(\frac{0,0,\pm h}{2})$	$\bar{\sigma}_2(\frac{0,0,\pm h}{2})$	$\bar{\sigma}_{13}(\frac{a}{2},0,0)$	$\bar{\sigma}_{23}(0,\frac{b}{2},0)$	$\bar{\sigma}_{12}(\frac{a}{2},\frac{b}{2},\pm\frac{h}{2})$
4	**Exact: Pagano**		1.556 -1.512	0.2595 -0.2533	0.239	0.1072	-0.1437 0.1481
	Present: FEM						
	2x2x3	8.588	1.514 -1.570	0.2733 -0.337	0.2125	0.07811	-0.1233 0.1621
10	**Exact: Pagano**		1.153 -1.152	0.1104 -0.1099	0.300	0.0527	-0.0707 0.0717
	Present: FEM						
	2x2x3	2.3926	1.097 -1.103	0.1112 -0.1211	0.2647	0.03438	-0.06109 0.06718
100	**Exact: Pagano**		±1.098	±0.0550	0.324	0.0297	∓0.0437
	Pesent: FEM						
	2x2x3	0.87133	±1.015	0.05092 -0.05102	0.2829	0.01828	-0.03629 0.03635
	Classical Plate Theory		±1.097	±0.0543	0.324	0.0295	∓0.0433

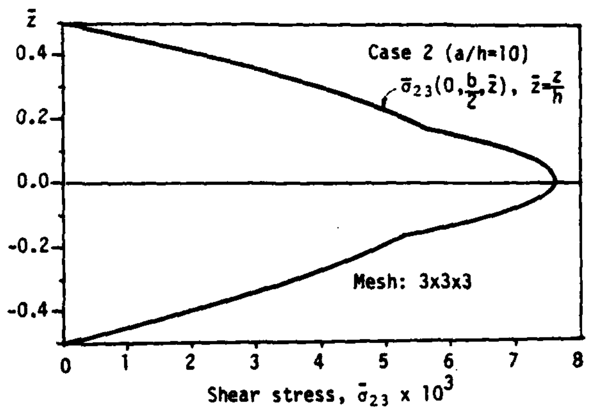


Figure 2. Distribution of the shear stress along $\bar{z}$

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER: VPI-E-82.19 — 2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): THREE-DIMENSIONAL FINITE-ELEMENT ANALYSIS OF LAYERED COMPOSITE STRUCTURES	5. TYPE OF REPORT & PERIOD COVERED: Interim 6. PERFORMING ORG. REPORT NUMBER: Tech. Report No. 29
7. AUTHOR(s): W.C. Chao, N.S. Putcha, and J.N. Reddy	8. CONTRACT OR GRANT NUMBER(s): N00014-78-C-0647
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Virginia Polytechnic Institute (subcontractor) University of Oklahoma	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NR 064-609 Division of Engineering Mechanics (structures)
11. CONTROLLING OFFICE NAME AND ADDRESS: Department of the Navy, Office of Naval Research Division of Engineering Mechanics Arlington, Virginia 22217	12. REPORT DATE: July, 1982 13. NUMBER OF PAGES: 12
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified 15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

This document has been approved for public release and sale; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Both parts of the report will appear in an Aerospace Division volume of ASME, ADVANCES IN AEROSPACE STRUCTURES AND MATERIALS. The papers will be presented at the 1982 Winter Annual Meeting of ASME, Phoenix, Arizona.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Composite materials, degenerate three-dimensional element, finite-element analysis, laminates, plates, shells, three-dimensional theory of elasticity.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Part 1 of the report is concerned with the large deformation bending of laminated composite shells by the "degenerate" three-dimensional element. The present results agree well with the results available in the literature. Part 2 of the report is concerned with the full three-dimensional finite-element analysis of laminated anisotropic composite plates. The present linear analysis results compare favorably with those presented by Pagano.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

DATE
ILMED
8